**Trend Series Analysis**

1. There are four important bases for classification of data namely qualitative, quantitative, geographical and chronological. In the classification on chronological basis, the data are arranged by successive time periods, e.g., years, quarters, months, etc. An arrangement of data by successive time periods is called a *‘time series’*.
2. A time series is a sequence of observations obtained as successive time periods.
3. Examples of time series are:
* Annual yield of a crop in a country for a number of years,
* Annual profit before tax of a firm,
* Daily temperature of a city,
* Annual rainfall of a country,
* Total monthly sales receipts in a departmental store,
* Daily closing prices of a share on the stock exchange,
* Annual balance of trade of a country,
* Weekly consumer price index number, etc.
1. Basically, there are two objectives of time series:
	1. to examine the patterns of change over time, and
	2. to use these patterns to forecast and predict future values.

***Graph of a Time Series – Histogram:***

A time series trend is graphically presented by plotting observed values against corresponding time points and joining these points by straight lines. This graph is commonly known as *‘Histogram’*.

**Example 1:**

Plot the following observed values on a graph:

# Table I – Quarterly Sales (In million Rs.)

|  |  |
| --- | --- |
| Time | **Sale** |
| 2003 | I | 219 |
|  | II | 357 |
|  | III | 645 |
|  | IV | 513 |
| 2004 | I | 549 |
|  | II | 640 |
|  | III | 701 |
|  | IV | 590 |
| 2005 | I | 657 |
|  | II | 394 |
|  | III | 543 |
|  | IV | 600 |

**Solution:**

***Signal and Noise:***

1. Time series may be considered as made up of two types of sequences, i.e., signal and noise. The sequence which follows some regular patterns of variation and can be completely determined or specified is called the *‘systematic sequence’* or *‘signal’*.
2. The sequence following random or irregular patterns of variation is called *‘noise’*.
3. Let the values of time series variable *Y* be *Y1*, *Y2*, ………., *Yn* observed at equal intervals of time *t1*, *t2*, …………, *tn*, then the time series may be represented by the model:

### Y = f(t) + U

Where *Y* : time series variable,

 *f(t)* : systematic sequence signal, and

 *U* : random sequence noise.

***Components of a Time Series:***

The change or variations in the observations of a time series are due to one or more of the four factors called *‘components of the time series’* or *‘characteristic movements of a time series’*. These components are:

1. Secular trend (T)
2. Seasonal variations (S)
3. Cyclical variations (C)
4. Random variations (R)
5. **Secular Trend (T):** The word *‘secular’* is used to mean *‘long-term’* or *‘relating to long periods of time’*. Thus, the secular trend refers to the movement of a time series in one direction over a fairly long period of time. The movement is smooth, steady and regular in nature. Such a movement characterises the general pattern of increase or decrease in an economic or social phenomenon.
6. **Seasonal Variations (S):** Such movements refer to short-term variations which a time series usually follows during corresponding months or seasons of successive years. It is refer to any variation of repeating nature, within a period of one year, caused by recurring events. For example, increased demand for woollen clothes during winter, increased sales at a departmental store before Eid, increased sale of candies before Christmas, etc.
7. **Cyclical Variations (C):** Such movements refer to long-term oscillations or swings about the trend line or curve. Since the movements take the form of upward and downward swings, they are also called *‘cycles’*. The movements are considered cyclical only if they recur after a period of more than one year. The term *‘cycle’* is used for *‘business cycle’*, which is consist of four phases, i.e., prosperity recession, depression and revival.
8. **Random Variations (R):** These movements refer to fluctuations of irregular nature caused by chance events such as war, flood, storm, earthquake, accidents, strikes, etc. They are also known as *‘irregular’*, *‘accidental’* or *‘erratic’* movements.

The first three components, i.e., secular trend, seasonal variations and cyclical variations, follow regular patterns of variation, therefore, fall under signal. While the random variations follow irregular patterns of movements, therefore, it falls under noise.

(a) Secular Trend

X

Y

(b) Secular Trend,

Cyclical and Seasonal Variations

X

Y

(c) Secular Trend and Cyclical Variations

X

Y

***Analysis of Time Series / Time Series Model:***

* 1. An approach to represent time series data is to multiply the four components . this is called *‘Multiplicative Time Series Model’*:

### Y = T × C × S × I

Where *Y* : observed value of a time series at a particular time point

 *T* : Trend (secular)

 *C* : Cyclical variations

 *S* : Seasonal variations

 *R* or *I* : Irregular or random variations

* 1. The second approach is based on additive law, known as *‘Additive Model’*:

### Y = T + C + S + R

* 1. This analysis is often called the *‘decomposition’* of a time series into basic component movements.

***Measurement of Secular Trend:***

1. Freehand curve method,
2. Semi-averages method,
3. Moving-averages method, and
4. Least-squares method.

Freehand, semi-averages and moving averages methods are used to study the pattern of change over a long period of time. They remove the short-term changes or smooth out the series. Least squares method is used to forecast the future values.

1. **Freehand Curve Method:** In this method the data are plotted on a graph measuring the time units (years, months, etc.) along the x-axis and the values of the time series variable along the y-axis. A trend line or curve is drawn through the graph in such a way that it shows the general tendency of the values.

**Example 2:**

Take the data from Example 1, and plot the observed on a graph and draw a trend line using *‘freehand curve method’*.

**Solution:**

 **2003 2004 2005**

1. **Semi-Averages Method:** In this method, the data are divided into two parts. If the number of values is odd, either the middle value is left out or the series is divided unevenly. The averages for each part are computed and placed against the centre of each part. The averages are plotted and joined by line. The line is extended to cover the whole data.

**Example 3:**

For the following data, calculate the trend values and plot them on a graph using *‘semi-averages method’*:

# Table II

**Pakistan’s Per Capita Income (In Rs.)**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | **1996** | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** |
| **PCI** | 15522 | 17393 | 18901 | 20377 | 25244 | 27227 | 28769 | 31572 | 35196 | 41008 |

**Solution:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **PCI** | Semi**Total** | **Semi****Average** | **Trend****Value** |
| 1996 | 15522 |  |  | 14180.6 |
| 1997 | 17393 |  |  | 16834.0 |
| 1998 | 18901 | 97437 | 19487.4 | 19487.4 |
| 1999 | 20377 |  |  | 22140.8 |
| 2000 | 25244 |  |  | 24794.2 |
| 2001 | 27227 |  |  | 27447.6 |
| 2002 | 28769 |  |  | 30101.0 |
| 2003 | 31572 | 163772 | 32754..4 | 32754.4 |
| 2004 | 35196 |  |  | 35407.8 |
| 2005 | 41008 |  |  | 38061.2 |

*Note:*

Difference between semi-averages: 32754.4 – 19487.4 = 13267

Average increase in income per year: 13267 ÷ 5 = 2653.4

Trend values for 1997: 19487.4 – 2653.4 = 16834

Trend values for 1999: 19487.4 + 2653.4 = 22140.8

1. **Moving-Averages Method:** Moving averages method is appropriate only when the trend is linear. It is also used to eliminate seasonal, cyclical and irregular fluctuations in the data. In this method, we find the simple average successively taking a specific number of values at a time. For e.g., if we want to find 3-year moving average, we shall find the average of the first three values, then drop the first value and include the fourth value. The process will be continued till all the values in the series are exhausted.

**Example 4:**

For the following data, calculate 3-year and 5-year moving averages and plot them on a graph using *‘moving-averages method’*:

**Annual GDP Growth Rate (in percent)**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| **%** | 6.6 | 1.7 | 3.5 | 4.2 | 3.9 | 1.8 | 3.1 | 4.8 | 6.4 | 8.4 |

**Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year** | **Values** | **3 year****moving****total** | **3 year****moving****average** | **5 year****moving****total** | **5 year****moving****average** |
| 1996 | 6.6 |  |  |  |  |
| 1997 | 1.7 | 11.8 | 3.93 |  |  |
| 1998 | 3.5 | 9.4 | 3.13 | 19.9 | 3.98 |
| 1999 | 4.2 | 11.6 | 3.87 | 15.1 | 3.02 |
| 2000 | 3.9 | 9.9 | 3.3 | 16.5 | 3.3 |
| 2001 | 1.8 | 8.8 | 2.93 | 17.8 | 3.56 |
| 2002 | 3.1 | 9.7 | 3.23 | 20 | 4 |
| 2003 | 4.8 | 14.3 | 4.77 | 24.5 | 4.9 |
| 2004 | 6.4 | 19.6 | 6.53 |  |  |
| 2005 | 8.4 |  |  |  |  |

1. **Least Squares Method:** The method of least squares states that of all the curves which can possibly be drawn to approximate the given data, the best fitting curve is the one for which the sum of squares of deviations is the least.

Method of least squares is used to fit a linear trend given by the straight line and non-linear trend given by a second and higher degree curves. In this method, an algebraic equation is fitted to the observed data. This equation may be linear, quadratic or exponental depending upon the pattern of time series graph.

Let  be the fitted equation, where  gives the estimated value of *y*. The difference  is called *‘error’*. Least square method finds the constants of the equation such that:

* sum of errors is zero, that is .
* sum of squared errors is minimum, that is 
1. **Fitting of Linear Equation:** If the graph of a time series show a linear trend, a linear equation is fitted:



Where  = the trend value for any time point,

 *a* & *b* = constants of the equation.

The normal equations are:

#### Σy = na + bΣx

and *Σxy = aΣx + bΣx2*

To reduce the computations involved in the solution of there normal equations, time-points are coded according to the following scheme, denoted by *x*:

For odd numbers of time-points, codes are:

…………….. –4, –3, –2, –1, 0, 1, 2, 3, 4, ………………….

For even numbers of time-points, codes are:

…………….. –7, –5, –3, –1, 1, 3, 5, 7, …………………….

In both cases, the sum of codes is zero.

Since the sum of codes (i.e., *Σx*) is zero, therefore the *Σx3* is also equal to zero, and the formulae of *a* and *b* are as follows:





**Example 5:**

For the following data, calculate the trend values and plot them on a graph using *‘least squares method’* by fitting linear equation:

**Table III**

**Pakistan’s GDP (In Trillion Rs.)**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| **GDP** | 3.10 | 3.15 | 3.26 | 3.40 | 3.53 | 3.59 | 3.71 | 3.89 | 4.14 | 4.48 |

**Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | **GDP(Y)** | X | XY | **X2** | **Trend****Y = 3.625 + 0.07118ּX** |
| 1996 | 3.10 | –9 | –27.9 | 81 | 2.984 |
| 1997 | 3.15 | –7 | –22.05 | 49 | 3.127 |
| 1998 | 3.26 | –5 | –16.3 | 25 | 3.269 |
| 1999 | 3.40 | –3 | –10.2 | 9 | 3.411 |
| 2000 | 3.53 | –1 | –3.53 | 1 | 3.554 |
| 2001 | 3.59 | 1 | 3.59 | 1 | 3.696 |
| 2002 | 3.71 | 3 | 11.13 | 9 | 3.839 |
| 2003 | 3.89 | 5 | 19.45 | 25 | 3.981 |
| 2004 | 4.14 | 7 | 28.98 | 49 | 4.123 |
| 2005 | 4.48 | 9 | 40.32 | 81 | 4.266 |
| Total | 36.25 | 0 | 23.49 | 330 |  |





The equation of the straight line is: *Y = a + bX*

The equation would be: *Y = 3.625 + 0.07118ּX*

1. **Fitting of Quadratic Equation:** The data may often be better expressed by a quadratic trend of the type:



which is a second degree curve called *‘parabola’*. Values of *a*, *b* and *c* are obtained by solving the normal equations by the principle of least squares:

*Σy = na + bΣx + cΣx2 = na + cΣx2*

*Σxy = aΣx + bΣx2 + cΣx3 = bΣx2*

*Σx2y = aΣx2 + bΣx3 + cΣx4 = aΣx2 + cΣx4*

**Example 6:**

For the following data, calculate the trend values and plot them on a graph using *‘least squares method’* by fitting quadratic equation:

**Table III**

**Pakistan’s GDP (In Trillion Rs.)**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| **GDP** | 3.10 | 3.15 | 3.26 | 3.40 | 3.53 | 3.59 | 3.71 | 3.89 | 4.14 | 4.48 |

**Solution:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | **GDP(Y)** | X | **XY** | **X2** | **X3** | **X4** | **X2Y** | **Trend** |
| 1996 | 3.10 | –9 | –27.9 | 81 | –729 | 6561 | 251.1 | 3.126 |
| 1997 | 3.15 | –7 | –22.05 | 49 | –343 | 2401 | 154.35 | 3.174 |
| 1998 | 3.26 | –5 | –16.3 | 25 | –125 | 625 | 81.5 | 3.246 |
| 1999 | 3.40 | –3 | –10.2 | 9 | –27 | 81 | 30.6 | 3.341 |
| 2000 | 3.53 | –1 | –3.53 | 1 | –1 | 1 | 3.53 | 3.460 |
| 2001 | 3.59 | 1 | 3.59 | 1 | 1 | 1 | 3.59 | 3.602 |
| 2002 | 3.71 | 3 | 11.13 | 9 | 27 | 81 | 33.39 | 3.768 |
| 2003 | 3.89 | 5 | 19.45 | 25 | 125 | 625 | 97.25 | 3.957 |
| 2004 | 4.14 | 7 | 28.98 | 49 | 343 | 2401 | 202.86 | 4.170 |
| 2005 | 4.48 | 9 | 40.32 | 81 | 729 | 6561 | 362.88 | 4.407 |
| Total | 36.25 | 0 | 23.49 | 330 | 0 | 19338 | 1221.05 |  |

*Σy = na + cΣx2*

*Σxy = bΣx2*

*Σx2y = aΣx2 + cΣx4*

Substituting the values:

*36.25 = 10a + 330c*

*23.49 = 330b*

*1221.05 = 330a + 19338c*

Solving the above equations, we get *a* = 3.528, *b* = 0.07118 and *c* = 0.00294

The equation of the fitted parabola is: 

Thus, the equation is: 

1. **Fitting of Exponental Equation:** Let the exponental equation be fitted is:



Taking log on both sides of the exponental equation:



or



Where 

A and B are computed using the two normal equations:



and converted to *a* and *b* by the relations:

*a = 10A* and *b = 10B*

**Example 7:**

Fit an exponental curve to the following data, using method of least squares:

**Pakistan Cotton Production (in thousand tonnes)**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| **Values** | 9374 | 9184 | 8790 | 11240 | 10732 | 10613 | 10211 | 10048 | 14265 |

**Solution:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Year** | ***x*** | ***y*** | **log *y*** | ***x*ּlog *y*** | ***x2*** | **Trend** |
| 1997 | –4 | 9374 | 3.9719 | –15.8876 | 16 | 8980 |
| 1998 | –3 | 9184 | 3.9630 | –11.889 | 9 | 9315 |
| 1999 | –2 | 8790 | 3.9440 | –7.888 | 4 | 9663 |
| 2000 | –1 | 11240 | 4.0508 | –4.0508 | 1 | 10023 |
| 2001 | 0 | 10732 | 4.0307 | 0 | 0 | 10397 |
| 2002 | 1 | 10613 | 4.0258 | 4.0258 | 1 | 10785 |
| 2003 | 2 | 10211 | 4.0091 | 8.0182 | 4 | 11187 |
| 2004 | 3 | 10048 | 4.0021 | 12.0063 | 9 | 11604 |
| 2005 | 4 | 14265 | 4.1543 | 16.6172 | 16 | 12037 |
| Total | 0 |  | 36.1517 | 0.9521 | 60 |  |

Normal equations are:



Substituting the values:

*36.1517 = 9A → A = 4.0169*

*0.9521 = 60B → B = 0.0159*

Now

*a* = 10A = 104.0169 = 10397

*b* = 10B = 100.0159 = 1.0373

The exponental curve is: 

Now, substituting the values: 

***Measurement of Seasonal Trend:***

* 1. Simple Average Method,
	2. Link Relative Method,
	3. Ratio to Moving Average Method, and
	4. Ratio to Trend Method.
1. **Simple Average Method:** Under this method, the average () of all the monthly or quarterly values for each year are found out. Each monthly or quarterly value (*yi*) is divided by the corresponding average and the results are expressed as percentage:



Then the mean index or seasonal index (*Si*) is calculated for each month or quarter. If the mean of all seasonal indices is not equal to 100, then they will be adjusted.

**Example 8:**

Take data from Example 1, and calculate the four seasonal indices by the *‘simple-average method’.*

**Solution:**

|  |  |  |
| --- | --- | --- |
| Year/Quarter | **y** | **Mean**  |
| **I** | **II** | **III** | **IV** |
| 2003 | 219 | 357 | 645 | 513 | 433.5 |
| 2004 | 549 | 640 | 701 | 590 | 620 |
| 2005 | 657 | 394 | 543 | 600 | 548.5 |

Now the above observed values are converted to indices using the following formula:



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year/Quarter** | **I** | **II** | **III** | **IV** | **Total** |
| 2003 | 50.52% | 82.35% | 148.79% | 118.34% |  |
| 2004 | 88.55 | 103.23 | 113.06 | 95.16 |  |
| 2005 | 119.78 | 71.83 | 99.00 | 109.39 |  |
| Season Index (*Si*) | 86.28% | 85.80% | 120.28% | 107.63% | 400.00 |

1. **Link Relative Method:** Under this method, the data for each month or quarter are expressed in percentage, known as *‘Link Relatives’*. An appropriate average of the link relatives is taken, usually a median is taken. Convert these averages into a series of chain indices. The chain indices are adjusted for the fraction of the effect of the trend. The adjusted chain indices are further reduced to the same level as the first month or quarter.

**Example 9:**

Take the data from Example 1, and calculate seasonal indices by using *‘link relative method’.*

**Solution:**

The observed values are converted into price relatives or link relatives using the following formula:

|  |  |
| --- | --- |
|  | Where Pn is the value of current year Po is the value of base year |

and then the link relatives are converted into chain indices (chaining process) using the following formula:

|  |  |
| --- | --- |
| = *(L.R. × C.I. of preceding year) ÷ 100* | Where L.R. is the link relative C.I. is the chain index |

|  |  |  |
| --- | --- | --- |
| Year | Quarter | Total |
| **I** | II | **III** | **IV** |
| 2003 | – | 163.01% | 180.67% | 79.53% |  |
| 2004 | 107.02% | 116.58 | 109.53 | 84.17 |  |
| 2005 | 111.36 | 59.97 | 137.82 | 110.50 |  |
| Median(link relative) | 109.19 | 116.58 | 137.82 | 84.17 |  |
| Chain Index | 100 | 116.58 | 160.67 | 135.24 | 512.49 |
| Adj. C. I. | 78.05%\* | 90.99% | 125.40% | 105.56% | 400.00 |

\* Adjusted chain index for QI: 100 ÷ 512.49 × 400 = 78.05, and so for other quarters.

1. **Ratio to Moving Average Method:** A 12-month or 4-quarter moving average centred is computed. The observed values are divided by the corresponding centred moving average and the results are expressed in percentage:

****

The monthly or quarterly averages of these percentages are found out. The adjusted values are the indices of the seasonal variations.

**Example 10:**

Take data from Example 1 and calculate seasonal indices using *‘ratio to moving average method’.*

**Solution:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Quarter** | ***y*** | **4-Quarter****Moving****Total** | **8-Quarter****Moving****Total** | **4-Quarter****Moving**Average**(Centred)** | **Ratio to Moving Average** |
| **I** | **II** | **III** | **IV** |
| 2003 | I | 219 |  |  |  | – |  |  |  |
|  | II | 357 |  |  |  | – |  |  |
| 1734 |
|  | III | 645 | 3798 | 475 |  |  | 135.8 |  |
| 2064 |
|  | IV | 513 | 4411 | 551 |  |  |  | 93.1 |
| 2347 |
| 2004 | I | 549 | 4750 | 594 | 92.4 |  |  |  |
| 2403 |
|  | II | 640 | 4883 | 610 |  | 104.9 |  |  |
| 2480 |
|  | III | 701 | 5068 | 634 |  |  | 110.6 |  |
| 2588 |
|  | IV | 590 | 4930 | 616 |  |  |  | 95.8 |
| 2342 |
| 2005 | I | 657 | 4526 | 566 | 116.1 |  |  |  |
| 2184 |
|  | II | 394 | 4378 | 547 |  | 72.0 |  |  |
| 2194 |
|  | III | 543 |  |  |  |  | – |  |
|  |
|  | IV | 600 |  |  |  |  |  | – |
|  |
| Mean Seasonal Index *(total = 410.5)* | 104.3 | 88.5 | 123.2 | 94.5 |
| Adjusted Seasonal Index (*Si*) *(total = 400)* | 101.6%\* | 86.2% | 120.1% | 92.1% |

\* Adjusted seasonal index for QI: 104.3 ÷ 410.5 × 400, and so on for other three quarters.

1. **Ratio to Trend Method:** An average for each year is found out and a straight line is fitted by least squares method. The trend values for each month or quarter are calculated on the assumption that the data correspond to the middle of the month or quarter. Each original value is divided by the corresponding calculated trend values and expressed in percentage. A mean of these percentages are calculated for each month or quarter. The adjusted values are the indices of seasonal variation.

**Example 11:**

Take data from Example 1 and calculate seasonal indices using *‘ratio to trend method’*.

**Solution:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Quarters** | ***y*** | ***x*** |  | ***x*** |  | ***x2*** |  |  |
| 2003 | I | 219 | –11 |  |  |  |  | 455 | 48.13% |
|  | II | 357 | –9 | 469 | 76.12 |
| 433.5 | –8 | –3468 | 64 |
|  | III | 645 | –7 | 484 | 133.26 |
|  |  |  |  |
|  | IV | 513 | –5 | 498 | 103.01 |
|  |  |  |  |
| 2004 | I | 549 | –3 | 512 | 107.23 |
|  |  |  |  |
|  | II | 640 | –1 | 527 | 121.44 |
| 620 | 0 | 0 | 0 |
|  | III | 701 | 1 | 541 | 129.57 |
|  |  |  |  |
|  | IV | 590 | 3 | 556 | 106.12 |
|  |  |  |  |
| 2005 | I | 657 | 5 | 570 | 115.26 |
|  |  |  |  |
|  | II | 394 | 7 | 584 | 67.47 |
| 548.5 | 8 | 4388 | 64 |
|  | III | 543 | 9 | 599 | 90.65 |
|  |  |  |  |
|  | IV | 600 | 11 | 613 | 97.88 |
|  |  |  | 0 | 1602 | 0 | 920 | 128 |  |  |





Now arranging the above calculated values in last column as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Year/Quarter** | **I** | **II** | **III** | **IV** | **Total** |
| 2003 | 48.13 | 76.12 | 133.26 | 103.01 |  |
| 2004 | 107.23 | 121.44 | 129.57 | 106.12 |  |
| 2005 | 115.26 | 67.47 | 90.65 | 97.88 |  |
| Seasonal Index (Si) | 90.21 | 88.34 | 117.83 | 102.34 | 398.72 |
| Adj. S.I. | 90.50%\* | 88.62% | 118.21% | 102.67% | 400.00 |

\* Adjusted seasonal index for QI: 90.21 ÷ 398.72 × 400, and so on.

***Measurement of Cyclical Variation:***

The cyclical and random components of a time series are first isolated from the time series using the multiplicative model:

*yi = Ti + Si + Ci + Ri*

Where *Ti*: Secular trend

 *Si*: Seasonal variation

 *Ci*: Cyclical variation

 *Ri*: Random variation

This can be done by dividing *yi* by the product of *Ti* and *Si*:



The Random component *Ri* will now be separated from the time series by using the smoothing technique, moving average. These moving averages show the indices of cyclical variation.

**Example 12:**

Take data from Example 1 and isolate cyclical component from the time series.

**Solution:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Quarters | ***y*** | **\*** | ***Si*\*\*** |  | ***Ci × Ri* =****×100** | **3-Quarter****Moving****Average****(*Ci*)** |
| 2003 | I | 219 | 455 | 86.28 | 392.57 | 55.79% | – |
|  | II | 357 | 469 | 85.80 | 402.40 | 88.72 | 85.10 |
|  | III | 645 | 484 | 120.28 | 582.16 | 110.79 | 98.41 |
|  | IV | 513 | 498 | 107.63 | 536.00 | 95.71 | 110.26 |
| 2004 | I | 549 | 512 | 86.28 | 441.75 | 124.28 | 120.51 |
|  | II | 640 | 527 | 85.80 | 452.17 | 141.54 | 124.52 |
|  | III | 701 | 541 | 120.28 | 650.71 | 107.73 | 115.95 |
|  | IV | 590 | 556 | 107.63 | 598.42 | 98.59 | 113.30 |
| 2005 | I | 657 | 570 | 86.28 | 491.80 | 133.59 | 103.60 |
|  | II | 394 | 584 | 85.80 | 501.07 | 78.63 | 95.86 |
|  | III | 543 | 599 | 120.28 | 720.48 | 75.37 | 81.74 |
|  | IV | 600 | 613 | 107.63 | 657.77 | 91.22 | – |

\* As calculated in the previous example.

\*\* As calculated in Example