Measures of the Shape of Distribution

The frequency distribution can be described by the following four characteristics:

1. The *‘central value’* in the distribution around which the observations tend to lie, which is described by the *‘measures of central tendency’*,
2. The *‘dispersion’* i.e., the extent to which the observations are spread out from the central value, which is described by the *‘measures of dispersion’*,
3. The manner in which the observations are distributed around the central values, i.e., whether the distributions is *‘symmetrical’* or *‘skewed’*, which is described by the *‘measures of skewness’*, and
4. The *‘peakedness’* or *‘flatness’* of the distribution which is measured relative to a distribution known as *‘normal distribution’*, which is described by the *‘measures of kurtosis’.*

All these four characteristics can be described by what are known as *‘moments’*.

***Moments:***

Moments are the AM of the powers to which the deviations are raised. Thus, the mean of the first power of the deviations from mean is the *‘first moment about mean’*, the mean of the second power (squares) of the deviations from mean is the *‘second moment about mean’*, and so on.

**(a) Moments about Mean:**

Symbolically, the first four *‘moments about mean’* (denoted by m1, m2, m3 and m4) are defined as:

|  |  |
| --- | --- |
| **Grouped Data** | **Ungrouped Data** |
|  |  |

**(b) Moments about an Arbitrary Value A:**

The first four *‘moments about an arbitrary value A’* are defined as follows: (Raw moment)

|  |  |
| --- | --- |
| **Grouped Data** | **Ungrouped Data** |
|  |  |

**(c) Moments about Zero / Origin:**

The first four *‘moments about zero / origin’* are defined as follows: (Raw moment)

|  |  |
| --- | --- |
| **Grouped Data** | **Ungrouped Data** |
|  |  |

It should be noted here that in *‘moments about zero’*, the arbitrary value is assumed to be 0. Further, the 1st moment about 0 is the AM.

**Example:**

|  |  |
| --- | --- |
| Class Interval | **Frequency** |
| 10-19 | 5 |
| 20-29 | 8 |
| 30-39 | 13 |
| 40-49 | 19 |
| 50-59 | 23 |
| 60-69 | 15 |
| 70-79 | 7 |
| 80-89 | 5 |
| 90-99 | 3 |
| 100-109 | 2 |
| Total | 100 |

Calculate:

1. Moments about origin
2. Moments about mean
3. Moments about the value 4

**Solution:**

1. **Moments about origin:**

*(See next page)*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C.I | ***f*** | ***x*** | ***fx*** | ***x2*** | ***fx2*** | ***x3*** | ***fx3*** | ***x4*** | ***fx4*** |
| 10-19 | 5 | 14.5 | 72.5 | 210.25 | 1051.25 | 3048.62 | 15243.12 | 44205.06 | 221025.31 |
| 20-29 | 8 | 24.5 | 196 | 600.25 | 4802 | 14706.13 | 117649 | 360300.06 | 2882400.5 |
| 30-39 | 13 | 34.5 | 448.5 | 1190.25 | 15473.25 | 41063.62 | 533827.13 | 1416695.06 | 18417035.81 |
| 40-49 | 19 | 44.5 | 845.5 | 1980.25 | 37624.75 | 88121.13 | 1674301.37 | 3921390.06 | 74506411.19 |
| 50-59 | 23 | 54.5 | 1253.5 | 2970.25 | 68315.75 | 161878.62 | 3723208.38 | 8822385.06 | 202914856.44 |
| 60-69 | 15 | 64.5 | 967.5 | 4160.25 | 62403.75 | 268336.13 | 4025041.87 | 17307680.06 | 259615200.94 |
| 70-79 | 7 | 74.5 | 521.5 | 5550.25 | 38851.75 | 413493.62 | 2894455.38 | 30805275.06 | 215636925.44 |
| 80-89 | 5 | 84.5 | 422.5 | 7140.25 | 35701.25 | 603351.13 | 3016755.62 | 50983170.06 | 254915850.31 |
| 90-99 | 3 | 94.5 | 283.5 | 8930.25 | 26790.75 | 843908.62 | 2531725.88 | 79749365.06 | 239248095.19 |
| 100-109 | 2 | 104.5 | 209 | 10920.25 | 21840.5 | 1141166.13 | 2282332.25 | 119251860.06 | 238503720.12 |
| Total | 100 |  | 5220 |  | 312855 |  | 20814540 |  | 1506861521.25 |



1. **Moments about mean:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C.I | ***f*** | ***x*** |  |  |  |  |  |  |  |  |
| 10-19 | 5 | 14.5 | -37.7 | -188.5 | 1421.29 | 7106.45 | -3048.62 | -15243.1 | 2020065.26 | 10100326.3 |
| 20-29 | 8 | 24.5 | -27.7 | -221.6 | 767.29 | 6138.32 | -14706.13 | -117649.04 | 588733.94 | 4709871.52 |
| 30-39 | 13 | 34.5 | -17.7 | -230.1 | 313.29 | 4072.77 | -41063.62 | -533827.06 | 98150.62 | 1275958.06 |
| 40-49 | 19 | 44.5 | -7.7 | -146.3 | 59.29 | 1126.51 | -88121.13 | -1674301.47 | 3515.30 | 66790.7 |
| 50-59 | 23 | 54.5 | 2.3 | 52.9 | 5.29 | 121.67 | 161878.62 | 3723208.26 | 27.98 | 643.54 |
| 60-69 | 15 | 64.5 | 12.3 | 184.5 | 151.29 | 2269.35 | 268336.13 | 4025041.95 | 22888.66 | 343329.9 |
| 70-79 | 7 | 74.5 | 22.3 | 156.1 | 497.29 | 3481.03 | 413493.62 | 2894455.34 | 247297.34 | 1731081.38 |
| 80-89 | 5 | 84.5 | 32.3 | 161.5 | 1043.29 | 5216.45 | 603351.13 | 3016755.65 | 1088454.02 | 542270.1 |
| 90-99 | 3 | 94.5 | 42.3 | 126.9 | 1789.29 | 5367.87 | 84908.62 | 254725.86 | 3201558.70 | 9604676.1 |
| 100-109 | 2 | 104.5 | 52.3 | 104.6 | 2735.29 | 5470.58 | 1141166.13 | 2282332.26 | 7481811.38 | 14963622.76 |
| Total | 100 |  |  | 0 |  | 40371 |  | 13855498.65 |  | 43338570.36 |

 Where 

1. **Moments about the value 4:**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| C.I | ***f*** | ***x*** | ***D =***  ***(x – A)*** | ***fD*** | ***D2 =***  ***(x – A)2*** | ***fD2*** | ***D3 =***  ***(x – A)3*** | ***fD3*** | ***D4 =***  ***(x – A)4*** | ***fD4*** |
| 10-19 | 5 | 14.5 | 10.5 | 52.5 | 110.25 | 551.25 | 1157.62 | 5788.1 | 12155.06 | 60775.3 |
| 20-29 | 8 | 24.5 | 20.5 | 164 | 420.25 | 3362 | 8615.12 | 68920.96 | 176610.06 | 1412880.48 |
| 30-39 | 13 | 34.5 | 30.5 | 396.5 | 930.25 | 12093.25 | 28672.62 | 672744.06 | 865365.06 | 11249745.78 |
| 40-49 | 19 | 44.5 | 40.5 | 364.5 | 1640.25 | 31164.75 | 66430.12 | 1262172.28 | 2690420.06 | 51117981.14 |
| 50-59 | 23 | 54.5 | 50.5 | 1161.5 | 2550.25 | 58655.75 | 128787.62 | 2962115.26 | 6503775.06 | 149586826.38 |
| 60-69 | 15 | 64.5 | 60.5 | 907.5 | 3660.25 | 54903.75 | 221445.12 | 3321676.8 | 13397430.06 | 200961450.9 |
| 70-79 | 7 | 74.5 | 70.5 | 493.5 | 4970.25 | 34791.75 | 350402.62 | 2452818.34 | 24703385.06 | 172923695.42 |
| 80-89 | 5 | 84.5 | 80.5 | 402.5 | 6480.25 | 32401.25 | 521660.12 | 2608300.6 | 41993640.06 | 209968200.3 |
| 90-99 | 3 | 94.5 | 90.5 | 271.5 | 8190.25 | 24570.75 | 741217.62 | 2223652.86 | 67080195.06 | 201240585.18 |
| 100-109 | 2 | 104.5 | 100.5 | 201 | 10100.25 | 20200.5 | 1015075.12 | 2030150.24 | 102015050.06 | 204030100.12 |
| Total | 100 |  |  | 4415 |  | 2272495 |  | 17608339.5 |  | 1202552241 |



***Symmetry and Skewness:***

1. A frequency distribution is said to be symmetrical if the values equidistant from a central maximum have the same frequency.
2. In a symmetrical distribution, a deviation below the mean is equal to the corresponding deviation above the mean. This is called *‘symmetry’*.
3. *‘Skewness’* is the lack of *‘symmetry’* in a distribution around some central value, i.e., mean, median or mode. It is, thus, the *‘degree of asymmetry’.*
4. When a distribution departs from symmetry, the mean, median and mode are pulled apart and one tail becomes longer than the other.
5. If the frequency curve has a longer tail to the right, the distribution is to be positively skewed.
6. If the frequency curve has a longer tail to the left, the distribution is said to be negatively skewed:

  



Positively Skewed Distribution

  



Negatively Skewed Distribution

**Measures of Skewness:**

1. To measure skewness is to measure the extent to which and also the direction in which the distribution (or curve) is non-symmetrical or skewed.
2. In a symmetrical distribution, the mean, median and mode coincide. In a skewed distribution, these values pulled apart.
3. There three measure of skewness:
   1. Absolute measure of skewness, i.e., the difference between the mean and mode. In a moderately skewed distribution, the empirical relation between , ,  is:



Therefore  or 

* 1. The difference between the distances (or differences) of Q3 and Q2, and Q2 and Q1:



or



* 1. The third order moment about mean:

 or 

**Relative Measures of Skewness:**

There are three relative measures of skewness:

1. **Pearson’s 1st and 2nd Coefficients of Skewness:**
   1. If we divide the absolute measure of skewness (i.e., the difference between  and  by SD, we get a relative measure of skewness:



* 1. If we employ the empirical relation between ,  and , we get the following alternate formula:



* 1. If the above formulae give positive results, it means the distribution is positively skewed and vice versa. For a symmetrical distribution, the measure will be equal to 0.

1. **Bowley’s Measure of Skewness:**
   1. This measure of skewness is true for a symmetrical distribution. In a symmetrical distribution, the quartiles (i.e., Q1, Q2, Q3 and Q4) are equidistant from the median, i.e., (Q3 – Q2) = (Q2 – Q1):



* 1. This measure will always be equal to zero for a symmetrical distribution. It is positive for positively skewed distribution and negative for negatively skewed distribution. This measure varies between – 1 to + 1.

1. **Moment Coefficient of Skewness:**
   1. In a symmetrical distribution, the sum of odd powers of deviations from mean is zero. Thus, the odd order moments about mean, i.e., m1, m3, etc., in a symmetrical distribution are zero.
   2. This measure of skewness is true in case of a skewed distribution:



* 1. Or, alternatively:



For a symmetrical distribution α3 and β1 will be 0.

**Example:**

|  |  |  |  |
| --- | --- | --- | --- |
| C.I | ***f*** | ***x*** | ***fx*** |
| 10-19 | 5 | 14.5 | 72.5 |
| 20-29 | 8 | 24.5 | 196 |
| 30-39 | 13 | 34.5 | 448.5 |
| 40-49 | 19 | 44.5 | 845.5 |
| 50-59 | 23 | 54.5 | 1253.5 |
| 60-69 | 15 | 64.5 | 967.5 |
| 70-79 | 7 | 74.5 | 521.5 |
| 80-89 | 5 | 84.5 | 422.5 |
| 90-99 | 3 | 94.5 | 283.5 |
| 100-109 | 2 | 104.5 | 209 |
| Total | 100 |  | 5220 |

Calculate:

1. Pearson’s first and second coefficient of skewness,
2. Quartile coefficient of skewness, and
3. Moment coefficient of skewness.

**Solution:**

1. **Pearson’s first and second coefficient of skewness:**

 ------------- Pearson’s first coefficient of skewness

 --- Pearson’s second coefficient of skewness

Comments: *Negatively skewed distribution.*

1. **Quartile coefficient of skewness:**



Comments: *Asymmetrical distribution – negatively skewed distribution*.

1. **Moment coefficient of skewness:**



***Kurtosis:***

1. Kurtosis is the degree of peakedness of a distribution usually taken relative to a normal distribution.
2. A distribution having a relatively high peak is called *‘leptokurtic’*.
3. A distribution which plat topped is called *‘platykurtic’*.
4. A normal distribution which is neither very peaked nor very flat-topped is also called *‘mesokurtic’*.

*μ x*

Leptokurtic (β2>3)

Mesokurtic (β2=3)

Platykurtic (β2<3)

*f(x)*

**Measures of Kurtosis:**

1. Two frequency distribution both symmetrical having same means and SDs may be different in *‘flatness’* of the top of their curves. The flatness of the top of a frequency curve is called the *‘kurtosis’*.
2. It is measured by a quantity denoted by β2 where:



1. If β2 = 3, it is mesokurtic or normal,

If β2 > 3, it is leptokurtic, and

If β2 < 3, it platykurtic.

Note: μ, β and α are used for population data, and , b and a for sample data.

**Example:**

Given *(moments about mean)*: m1 = 0, m2 = 1.25, m3 = 0.267, m4 = 2.264

Find the coefficient of kurtosis β2 and comment on the flatness of the distribution.

**Solution:**



Since β2 is less than 3, therefore, the distribution is Platykurtic.

***Relation between Moments:***



or



We have 

Since , 

and 

**Example:**

Convert the following moments about origin into moment about mean:



**Solution:**

