Index Numbers

***Introduction:***

1. An index number is a device which shows by its variations the change in a magnitude which is not capable of accurate measurement in itself or of direct valuation over time.
2. To measure changes in a situation we combine the prices and qualities and find a single number. This single number which shows overall changes in a phenomenon is known as ‘Index Number’.
3. It is used to compare changes in a complex phenomenon like the cost of living, total industrial production, wages, etc.
4. It is very useful in measuring changes in prices and quantities of commodities with different measuring units, for example, wheat per maund, cloth per yard, etc., which cannot be compared directly.

***Types of Index Number:***

* 1. **Price Index Number:** It compares changes in prices, from one period to another. Wholesale price index and cost of living index are the examples.
	2. **Quantity Index Number:** It measures how much the quantity of a variable changes over time. Index of industrial production and business activity index are examples.
	3. **Value Index Number:** It measures changes in total monetary worth. It combines price and quantity changes to present a more informative index. Index of GNP and index of retail sales are the examples.



***Uses of Index Numbers:***

1. An index number is a device for measuring changes in a variable or a group of related variables.
2. It can be used to compare changes in one or more variables in one period with those of others or in one region with those in the others.
3. The index number of industrial activity enables us to study the progress of industrialisation in the country.
4. The quantity index numbers show rise or fall in the volume of production, volume of exports and imports, etc.
5. The cost of living index numbers are, in fact, the retail price indices. They show changes in the prices of goods generally consumed by the people. Therefore, they can help the government to formulate the suitable price policy.
6. The cost of living index number can be made a basis for regulation of wage rates and can be used by industrial and commercial organisations to grant dearness allowance and bonus to their employees in order to meet the increased cost of living.
7. Index numbers are also used for forecasting business activity and in discovering seasonal fluctuations and business cycles.

***Steps in the Construction of Index Numbers of Prices:***

* 1. Defining the purpose and scope of index number, i.e., the general-purpose or special purpose,
	2. Selecting commodities to be included,
	3. Collection of prices, i.e., (a) considering the prices to be used like average price, retail price or wholesale price, etc; and (b) the sources of price data like from representative markets, price lists or trade journals.
	4. Selecting base period, (a) fixed-base method, and (b) chain-base method.
	5. Choice of average to be used, i.e., AM, median or GM.
	6. Selecting suitable weights: (a) implicit weighting, and (b) explicit weighting.

**Notations:**

Pn = Price in current year

Po = Price in base year

Qn = Quantity in current year

Qo = Quantity in base year

Pon = Price for the nth year to the base year

Qon = Quantity for the nth year to the base year

***Construction of Price Index Numbers:***

1. Simple Relatives or Simple Index Numbers,
2. Unweighted Index Numbers, and
3. Weighted Index Numbers.
4. **Simple Relatives:** are further classified into two categories:
5. **Price Relatives:** are obtained by dividing the price in a given year by the base year price and expressed as percentage. Thus:



**Example:**

The prices of sugar for 2001 and 2005 are given as below:

|  |  |
| --- | --- |
| **Year** | **Price / Kg** |
| 2001 | 11 |
| 2005 | 30 |

**Required:**

1. Taking 2001 as base year, find price relative for 2005.
2. Taking 2005 as base year, find price relative for 2001.

**Solution:**

1. **Base year: 2001**

|  |  |  |
| --- | --- | --- |
| **Year** | Price | **Price Relative (v)** |
| 2005 | 30 |  |

1. **Base year: 2005**

|  |  |  |
| --- | --- | --- |
| Year | Price | **Price Relative (V)** |
| 2001 | 11 |  |

1. **Link Relatives:** are obtained by dividing the price in a given year by the price in the preceding year and expressed as percentage:



Link relatives are not directly comparable, therefore, they are converted to a fixed based index number. The process of conversion is called the ‘chaining process’, and the index numbers so obtained are chain indices:

= (L.R. × C.I. of preceding year) ÷ 100

**Example:**

The price of rice for the 6 years is as follows:

|  |  |
| --- | --- |
| Year | **Price / Kg** |
| 2000 | 21 |
| 2001 | 20 |
| 2002 | 20 |
| 2003 | 22 |
| 2004 | 25 |
| 2005 | 28 |

**Required:**

Taking 2000 as base year, find price relatives and chain indices for the years 2001 to 2005.

**Solution:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Year** | **Price** | **Price Relative (V)** | Chain Indices |
| 2000 | 21 |  | 100% |
| 2001 | 20 |  |  |
| 2002 | 20 |  |  |
| 2003 | 22 |  |  |
| 2004 | 25 |  |  |
| 2005 | 28 |  |  |

1. **Unweighted Index Numbers:** There are two methods of constructing this type of index:
2. **Simple Aggregative Method:** In this method, the total of the prices of commodities in a given year is expressed as percentage of the total of the prices of commodities in the base year:



This method has two disadvantages which make it unsatisfactory:

* It does not take into account the relative importance of various commodities.
* The units in which prices are given, e.g., maunds, yards, gallons, etc., affect the value of index very much.

**Example:**

The prices of 3 commodities for the 5 years are as follows:

|  |  |
| --- | --- |
| Commodity | **Prices (per kg)** |
| **2001** | **2002** | **2003** | **2004** | **2005** |
| Rice | 20 | 20 | 22 | 25 | 28 |
| Sugar | 11 | 12 | 14 | 27 | 30 |
| Tea | 178 | 176 | 174 | 180 | 180 |

**Required:**

Simple aggregative index numbers for the years 2001-05, with 2001 as base year.

**Solution:**

|  |  |
| --- | --- |
| **Commodity** | **Prices (per kg)** |
| **2001** | **2002** | **2003** | **2004** | **2005** |
| Rice | 20 | 20 | 22 | 25 | 28 |
| Sugar | 11 | 12 | 14 | 27 | 30 |
| Tea | 178 | 176 | 174 | 180 | 180 |
| Total | 209 | 208 | 210 | 232 | 238 |
| SimpleAggregativeIndex |  |  |  |  |  |

1. **Average of Relatives’ Method:** In this method, we use the average (mean, median, GM, etc.) of the price relatives or link relatives. It does not affect the value of index numbers. The only disadvantage of this method is that it gives equal weight to all commodities.

**Example:**

The prices of 3 commodities for the 5 years are as follows:

|  |  |
| --- | --- |
| Commodity | **Prices (per kg)** |
| **2001** | **2002** | **2003** | **2004** | **2005** |
| Rice | 20 | 20 | 22 | 25 | 28 |
| Sugar | 11 | 12 | 14 | 27 | 30 |
| Tea | 178 | 176 | 174 | 180 | 180 |

**Required:**

Construct price index numbers using average of relatives’ method, taking 2001 as base year.

**Solution:**

|  |  |
| --- | --- |
| Commodity | **Prices (per kg)** |
| **2001** | **2002** | **2003** | **2004** | **2005** |
| Rice |  |  |  |  |  |
| Sugar |  |  |  |  |  |
| Tea |  |  |  |  |  |
| Total | 300 | 307.97 | 335.02 | 471.52 | 513.85 |
| Mean(Index) | 100 | 102.66 | 111.67 | 157.17 | 171.28 |

1. **Weighted Index Numbers:** This type of index can be further classified into two categories:
2. **Weighted Aggregative Index Numbers:** In these index numbers, the quantities produced, sold or bought or consumed during the base year or current year are used as weights. These weights indicate the importance of the particular commodity. Some well-known weighted index numbers are given below:[[1]](#footnote-2)\*
	1. **Lespeyre’s Index:** This index uses base year quantities as weights. For this reason, it is also known as ‘Base Year Weighted Index’:



Here W = Qo

* 1. **Paasche’s Index:** This index uses current years quantity as weights. For this reason, it is known as ‘Current Year Weighted Index’:



Here W = Qn

* 1. **Fisher’s Ideal Index:** This index number is the GM of the Lespeyre’s and Paasche’s index numbers. It is called ‘ideal’ because it satisfies two tests (Time Reversal and Factor Reversal Tests):



* 1. **Marshall-Edgeworth’s Index:** This index number uses the average of the base year and current quantities as weights:



**Example:**

|  |  |  |
| --- | --- | --- |
| **Commodities** | **2001** | **2005** |
| **Price (Rs. / kg)** | **Qty. (kgs)** | **Price (Rs. / kg)** | **Qty. (kgs)** |
| Rice | 20 | 100 | 28 | 160 |
| Sugar | 11 | 18 | 30 | 37 |
| Salt | 1 | 1 | 5 | 1.1 |
| Milk | 18 | 57 | 32 | 149 |

**Required:**

Construct the following price index numbers using 2001 as base year:

1. Lespeyre’s
2. Paasche’s
3. Fisher’s
4. Marshall-Edgeworth’s

**Solution:**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **2001** | **2005** | **PoQo** | **PnQo** | **PnQn** | **PoQn** | **Qo+Qn** | **Po(Qo+Qn)** | **Pn(Qo+Qn)** |
| **Po** | **Qo** | **Pn** | **Qn** |
| Rice | 20 | 100 | 28 | 160 | 2000 | 2800 | 4480 | 3200 | 260 | 5200 | 7280 |
| Sugar | 11 | 18 | 30 | 37 | 198 | 540 | 1110 | 407 | 55 | 605 | 1650 |
| Salt | 1 | 1 | 5 | 1.1 | 1 | 5 | 5.5 | 1.1 | 2.1 | 2.1 | 10.5 |
| Milk | 18 | 57 | 32 | 149 | 1026 | 1824 | 4768 | 2682 | 206 | 3708 | 6592 |
| Total |  |  |  |  | 3225 | 5169 | 10363.5 | 6290.1 |  | 9515.1 | 15532.5 |

1. **Lespeyre’s:**



1. **Paasche’s:**



1. **Fisher’s:**



1. **Marshall-Edgeworth’s:**



1. **Weighted Average of Relatives:** The formula of weighted average of relatives is:



or

 (Arithmetic Mean taken as average); where 

or

 (Geometric Mean taken as average)

The total value of the commodity is used as weights. If the base year value (PoQo) is used as base, then the formula becomes:



or



If the current year value (PnQn) is used as base, then the formula becomes:



**Example:**

|  |  |  |
| --- | --- | --- |
| **Commodity** | **Prices** | Weights |
| **2001** | **2005** |
| Rice | 20 | 28 | 35 |
| Tea | 178 | 180 | 5 |
| Sugar | 11 | 30 | 24 |

**Required:**

Weighted index for 2005, taking 2001 as base year using:

1. Arithmetic Mean
2. Geometric Mean

**Solution:**

1. **Arithmetic Mean:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Commodity** | **V** | W | **VW** |
| Rice |  | 35 | 4900 |
| Tea |  | 5 | 505.6 |
| Sugar |  | 24 | 6545.52 |
|  |  | 64 | 11951.12 |



1. **Geometric Mean:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Commodity** | **V** | **W** | **log V** | **W.logV** |
| Rice |  | 35 | 2.146 | 75.11 |
| Tea |  | 5 | 2.005 | 10.025 |
| Sugar |  | 24 | 2.436 | 58.464 |
|  |  | 64 |  | 143.599 |



**Quantity Index Number:** The formula described for obtaining price indices can be easily used to obtain quantity indices or volume indices simply by interchanging the Ps and Qs, for example:



and:



The Lespeyre’s index number can be converted as follows:



and so on.

**Value Index Numbers:** Like price or quantity index numbers, we can obtain formulae for value index numbers. The simplest value index number is defined as below:



This is the *‘Simple Aggregative Index’* because the values have not been obtained.

***Theoretical Tests for Index Numbers:***

According to Dr. Irvin Fisher, a good index number is required to satisfy the theoretical tests given below:

1. **Time Reversal Test:** This test may be stated as follows:

“If the time subscripts of a price (or quantity) index number formula be interchanged, the resulting price (or quantity) index formula should be reciprocal of the original formula”

 or Pon × Pno = 1

The Fisher’s and Marshall-Edgeworth’s formulae satisfy the *‘Time Reversal Test’*.

1. Lets take ***Lespeyre’s*** formula and interchange the time subscripts:



Therefore, Lespeyre’s index does not satisfy the tests.

1. Now take ***Paasche’s*** formula and by interchanging the time subscripts, we get:



The above calculation shows that the Paasche’s formula does not satisfy the time reversal test.

1. The ***Fisher’s*** index formula is given below:



On interchanging the time subscripts, we get



Now, by multiplying the resulting formula to the original formula, we get:



It means that Fisher’s index number satisfies the time reversal test.

1. The time reversal test for ***Marshall-Edgeworth*** index is as follows:



It means that the time reversal test for Marshall-Edgeworth index is satisfied.

1. **Factor Reversal Test:** This test may be stated as follows:

“If the factors Ps and Qs occurring in a price (or quantity) index formula be interchanged so that a quantity (or price) index formula is obtained, then the product of the two index numbers should give the true value index number”



1. Lets take the ***Lespeyre’s*** formula and interchange the factors and then multiply the same with the resulting formula:



Therefore, the factor reversal test on Lespeyre’s formula is failed and it is concluded that Lespeyre’s index number is not a Value Index Number.

1. Now take ***Paasche’s*** formula and interchange the factors and then multiply both the formulae:



The above test shows that Paasche’s index number does not satisfy the factor reversal test and it means that Paasche’s index formula is not a value index formula.

1. For ***Fisher’s*** index:



Therefore, it is concluded that factor reversal test is also satisfied in case of Fisher’s index. Since the Fisher’s index satisfies the time reversal test and factor reversal test, it is called Fisher’s Ideal Number.

1. The factor reversal test is not satisfied for ***Marshall-Edgeworth*** index, as:



**Ideal Index Number:** The Fisher’s index number is also known ideal index number in this sense that other index number, i.e., Lespeyre’s and Paasche’s either overstate or understate expenditure index. The Lespeyre’s index number overstates the index and Paasche’s index number understates the index. The ‘true’ index lies somewhere between Lespeyre’s and Paasche’s indices, and Fisher has suggested that it is equivalent to the GM of the two, i.e.:



Therefore, Fisher’s index numbers are ideal in the sense that they are the only ones that correctly predict the expenditure index.

***Wholesale Price Index:***

The whole price index number is designed to measure changes in the goods and services produced in different sectors of the economy and traded in wholesale markets. These goods and services even include electricity, gas, petrol, telecommunication, etc. These indices are constructed by weighted aggregative method with quantities produced or sold as weights.

***Consumer Price Index (CPI):***

* 1. This index is also known as the *‘Cost of Living Index’* or *‘Retail Price Index’*. It is designed to measure changes in the cost of living.
	2. By cost of living, the cost of goods and services of daily use purchased by a particular class of people in a city or town is meant. These goods and services are known as *‘market basket’* consists of food, house rent, apparel, energy, education, health and miscellaneous items.
	3. CPI is essentially a weighted aggregative price index. The prices used are the coverage retail prices paid by the consumers for purchase of goods and services. The weights are proportion of expenditure on different goods and services.

**Construction of Consumer Price Index:**

Following steps are involved in the construction of CPI:

1. **Scope:** As the first step the scope of index number is determined, e.g., the industrial workers, middle class, salaried individuals, low-income earners, etc. It is, therefore, necessary to specify the class of people and the locality where they reside. The class or group of people considered should, as far as possible, be homogenous with regard to their incomes and consumption patterns.
2. **Family Budget Inquiry and Allocation of Weights:** The second step is to conduct a family budget inquiry so as to ascertain the proportions of expenditure on different items and to assign weights to various items. The information regarding the nature, quality and quantities of commodities consumed should be analysed and weights assigned in proportion to the expenditure on different items.
3. **Price Data:** The prices used in the construction of consumer price index are the retail prices.
4. **Methods of Construction:** Two methods are used for the construction of CPI. The index numbers under both methods are the same. These methods are:
5. **Aggregative Expenditure Method:** In this method, the quantities consumed in the base year are taken as weights. It is thus the base year weighted index number given by Lespeyre’s formula:



1. **Family Budget Method:** This method is the weighted average of relatives. The amounts of expenditure incurred by families on various items in the base period are used as weights. This method is known as ‘Family Budget Method’ because the amounts of money spent by the families are obtained from a family inquiry:

 Where  and W = PoQo

**Example:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Commodity** | **Qty.****Consumed** | **Unit of Price** | **Price** |
| **2001** | **2005** |
| Wheat | 250 kgs | Rs. per 10 kgs bag | 120 | 150 |
| Rice | 100 kgs | Rs. per 40 kgs *bori* (bag) | 800 | 1120 |
| Sugar | 18 kgs | Rs. per 40 kgs *bori* (bag) | 440 | 1200 |
| Milk | 57 kgs | Rs. per kg | 18 | 32 |
| Tea | 14 kgs | Rs. per 100 kgs bag | 17800 | 18000 |
| Salt | 1 kg | Rs. per kg | 1 | 5 |

Compute consumer price index number for the year 2005 taking 2001 as base year using:

1. Aggregative Expenditure Method, and
2. Family Budget Method.

**Solution:**

1. **Aggregative Expenditure Method:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Quantity****Consumed** | **Qo** | **Prices** | **Aggregate****Expenditure** |
| **Po** | **Pn** | **PoQo** | **PnQo** |
| Wheat | 250 kgs | 25 bags | 120 | 150 | 3000 | 3750 |
| Rice | 100 kgs | 2.5 *boris* (bags) | 800 | 1120 | 2000 | 2800 |
| Sugar | 18 kgs | 0.45 *bori* (bag) | 440 | 1200 | 198 | 540 |
| Milk | 57 kgs | 57 kgs | 18 | 32 | 1026 | 1824 |
| Tea | 14 kgs | 0.14 bag | 17800 | 18000 | 2492 | 2520 |
| Salt | 1 kg | 1 kg | 1 | 5 | 1 | 5 |
|  |  |  |  |  | 8717 | 11439 |



1. **Family Budget Method:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Qty.****Consumed** | **Qo** | **Po** | **Pn** | V | **W** | **WV** |
| Wheat | 250 kgs | 25 bags | 120 | 150 |  | 3000 | 375000 |
| Rice | 100 kgs | 2.5 *boris* (bags) | 800 | 1120 |  | 2000 | 280000 |
| Sugar | 18 kgs | 0.45 *bori* (bag) | 440 | 1200 |  | 198 | 54000.54 |
| Milk | 57 kgs | 57 kgs | 18 | 32 |  | 1026 | 182402.28 |
| Tea | 14 kgs | 0.14 bag | 17800 | 18000 |  | 2492 | 251991.04 |
| Salt | 1 kg | 1 kg | 1 | 5 |  | 1 | 500 |
|  |  |  |  |  |  | 8717 | 1143893.86 |



**Uses of Consumer Price Index:**

1. **Purchasing Power of Money:** The purchasing power of a rupee is the reciprocal of CPI. It expresses the purchasing power of rupee in a current time period relative to the base period:



1. **Deflation of Per Capita Income:** The effect of changing prices on per capita income may be removed by deflating the income expressed in current money by CPI to produce a measure expressed in terms of deflated (real) money. This relationship is:



The deflated (or real) per capital income is expressed in terms of the price level at the time of the base period of CPI.

**Example:**

For the following CPI, calculate the purchasing power of rupee for each year:

|  |  |
| --- | --- |
| **Year** | **CPI** |
| 2000 | 100 |
| 2001 | 103.54 |
| 2002 | 106.75 |
| 2003 | 111.63 |
| 2004 | 121.98 |

**Solution:**



|  |  |  |
| --- | --- | --- |
| **Year** | CPI | **Purchasing Power of Rupee** |
| 2000 | 100 | 1.0000 |
| 2001 | 103.54 | 0.9658 |
| 2002 | 106.75 | 0.9368 |
| 2003 | 111.63 | 0.8958 |
| 2004 | 121.98 | 0.8198 |

**Example:**

Deflate the Per Capita Income (PCI) by the consumer price index given in the following table, with base year 2000:

|  |  |  |
| --- | --- | --- |
| Year | CPI | **PCI****(In US$)** |
| 2000 | 100 | 526 |
| 2001 | 103.54 | 501 |
| 2002 | 106.75 | 503 |
| 2003 | 111.63 | 579 |
| 2004 | 121.98 | 657 |

**Solution:**

Deflated Per Capita Income:



|  |  |  |
| --- | --- | --- |
| **Year** | **CPI** | Per Capita Income |
| **Current** | **Real** |
| 2000 | 100 | 526 |  |
| 2001 | 103.54 | 501 |  |
| 2002 | 106.75 | 503 |  |
| 2003 | 111.63 | 579 |  |
| 2004 | 121.98 | 657 |  |

***Construction of Wholesale and Consumer Price Indices in Pakistan:***

Three price indices are prepared in Pakistan by Federal Bureau of Statistics:

1. Wholesale Price Index,
2. Consumer Price Index, and
3. Sensitive Price Indicator.
4. **Wholesale Price Index:** This index is based on 91 commodities comprising 690 specifications. Lespeyre’s method is used and the average wholesale prices for the current year are used.
5. **Consumer Price Index (CPI):** This index is based on around 500 items of daily use grouped in 9 groups. Prices are collected from 25 major cities of the country. Number of markets varied from 1 to around 15 depending upon the size of the town. The average prices collected from the markets of a town represent the average retail price of that town. Lespeyre’s method is used for construction of CPI.
6. **Sensitive Price Indicator:** This index measures changes in the retail prices of around 50 essential items of daily use of low-income earners’ group.

***Splicing Index Numbers:***

* 1. A series of index numbers may be discontinued because of obsolete commodities included in it or because of change in weights of these commodities.
	2. If a new series of index numbers is constructed with changed commodities or changed weights, the two series (old and new) are not comparable. The old and new series must therefore be adjusted so that the two series are comparable.
	3. To adjust the new series, new index numbers are multiplied by the ratio of the old to the new index in the period of discontinuation:



* 1. Likewise, to adjust the old series, old index numbers are multiplied by the ratio of the new to old index:



The above procedures are known as *‘Splicing Index Numbers’.*

**Example:**

In the data given below, 2001 is the year of discontinuation of the old series. Construct a continuous series by splicing:

1. Old series, and
2. New series.

|  |  |  |
| --- | --- | --- |
| **Year** | **Index****(Old Series)** | **Index****(New Series)** |
| 1995 | 99.8 |  |
| 1996 | 96.7 |  |
| 1997 | 95.3 |  |
| 1998 | 111.9 |  |
| 1999 | 134.6 |  |
| 2000 | 159.8 |  |
| 2001 | 173.2 | 96.7 |
| 2002 |  | 100.0 |
| 2003 |  | 100.9 |
| 2004 |  | 109.1 |
| 2005 |  | 111.0 |

**Solution:**

1. **Old Series:** To splice the old series, multiply old indices by 0.5583, i.e.,.
2. **New Series:** To splice the new series, multiply new indices by 1.7911, i.e., .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Year** | **Index****(Old)** | **Index****(New)** | **Spliced****Index****(Old)** | **Spliced****Index****(New)** |
| 1995 | 99.8 |  | 55.7 | 99.8 |
| 1996 | 96.7 |  | 54.0 | 96.7 |
| 1997 | 95.3 |  | 53.2 | 95.3 |
| 1998 | 111.9 |  | 62.5 | 111.9 |
| 1999 | 134.6 |  | 75.1 | 134.6 |
| 2000 | 159.8 |  | 89.2 | 159.8 |
| 2001 | 173.2 | 96.7 | 96.7 | 173.2 |
| 2002 |  | 100.0 | 100.0 | 179.1 |
| 2003 |  | 100.9 | 100.9 | 180.7 |
| 2004 |  | 109.1 | 109.1 | 195.4 |
| 2005 |  | 111.0 | 111.0 | 198.8 |

1. \* W.A.I.N. is equal to  [↑](#footnote-ref-2)