

ECONOMETRICS (PAPER- IVC)

M.A (FINAL) EXTERNAL ANNUAL EXAMINATION - 1997

Time Allowed: 3 Hours

Maximum Marks: 100

1) Attempt any four questions.

2) All question carry equal marks

1. a) Differentiate between a Normal Random Variable and a Standard Normal Random Variable.

b) The following table gives Bond Rating (x) and Yield to Maturity (Y)% of 50 Bonds.

(i) Convert this table into a table of joint p.d.fs.

(ii) Calculate the Marginal p.d.fs. of X and Y.

(iii) Compute $E(Y/X=1)$, $E(YIX=2)$, $E(X/Y=13)$ and $p(X=Z/Y=18)$.

(iv) Find out if X and Y are independent or not.

Y	X	1	2	3
9.00		13	5	0
13.5		2	12	4
18.5		0	3	11

2. Given the data on GNP and Public Expenditure (E) on Pakistan over a period of 1971-1980 (in million of Rupees in 1959-1960 Prices), set up the ANOVA table to test the hypothesis that change in GNP hand no effect on E.

	GNP	E
1971	32883	7549.93
1972	35360	7679.96
1973	38085	8622.93
1974	39651	10251.4
1975	41410	10648.9
1976	43022	10484.0
1977	47480	10818.0
1978	49953	12511.0
1979	53292	13343.3
1980	56237	12625.2

3. State with brief reasons, whether the following statements are True, False or uncertain:

(i) Multi-collinearity is harmless the objective of the analysis is predication only.

(ii) Ceteris paribus, the higher the VIF is the longer the variances of OLS estimates.

(iii) If Heteroscedasticity is present the conventional t and F tests are invalid.

(iv) The Durbin Watson d-test assumes that the variance of error term is Homoscedastic.

(v) The first difference transformation to eliminate auto correlation assumes that the coefficient of auto correlation ρ is -1 .

4. Discuss with help of suitable examples the possible uses of Dummy variables in

(i) Comparing two regressions

(ii) Deseasonalizing time series data

(iii) Pooling time-series with cross-section data

(iv) Piece wise liner regressing models.

5. Develop a simultaneous equations model for the demand and supply of Cigarettes in Pakistan. What type of biases and/or problems can you encounter while completing your task?

6. Discuss situations, consequences, detection and remedy for the presence of auto-correlation in a model.

7. a) Show that the least square estimator of β_2 in $Y = \beta_1 + \beta_2 X_2 + \mu$ Blue.

b) Develop a statistic to test the significance of an observe coefficient β_2 .

8. In a study of 89 firms, the dependent variable was total cost (X_1) with explanatory variables, rate of output (X_2) and rate of absenteeism(X_3).

The mean are $\bar{X}_1 = 5B$, $\bar{X}_2 = 2.9$ and $\bar{X}_3 = 3.9$ and the matrix showing sums of square and cross products adjusted for mean is

	X1	X2	X3
X1	113.6	36.8	39.1
X2		50.5	-66.2
X3			967.1

Estimate the relationship between X_1 and the other two variables.

Estimate the standard errors coefficients and test the significance of

β_2 and β_3

1. For the following random sample (Tale 1.1) of five. Find:

- The proportion of the variation in Y explained by its regression on X.
- The proportion unexplained.

Table 1.1

Y	X
10	8
15	10
0	6
20	12
20	9

2. Consider the following estimated equation.

$$\text{Income} = 1.315 + 0.797 (\text{Education}) \quad R^2 = 0.285$$

- Based on the regression reported, determine the predicted value of Income corresponding to Education = 12.
- Determine the change in predicted income associated with a change from Education = 12 to Education = 16.
- Interpret the estimated Value of the intercept.
- Suppose that Equation is the correct specification of the relation between Income and Education. If the regression explains 28.5 percent of the variation in Income in the sample, what accounts for the rest of the variation?

3. In a study of B9 firms, the dependent variable was total cost (Y) with explanatory variable, rate of output (x_2) and rate of absenteeism (x_3) while the means are 5.8, 2.9 and 3.9 respectively. The matrix showing sum of squares and cross product adjusted for means is as under:

	Y	x_2	x_3
y	113.6	36.8	39.1
x_2		50.5	-66.2
x_3			967.1

Estimate the coefficients of the model, find the value of adjusted R^2 , and compute the standard errors of coefficient and the t values for testing the significance.

4. Following regression was estimated with 10 observations:

$$Y_1 = a + bX_1 + e_1$$

I need the ANOVA table to perform some tests and interpret the result but my old computer printer gave the following point out with some valuable information missing. How I find these figures.

Analysis of Variance

Source	DF	Sum of Squares	Mean Squares	F-Value
Model	1	2674.05	7	?
Error	?	?	96.994	?
Total	?			

$R^2 = ?$
 $\bar{R}^2 = ?$

5. Consider the following demand and supply model for money.

Demand	$-M^d = b_1 + b_2 Y + b_3 R + b_4 P + a$
Supply	$M^s = a_1 + a_2 Y + e.$

Where M = money

Y = Income,

R = rate of interest (predetermined)

P = price (predetermined)

- Is the demand function identified?
 - Is the supply function identified?
 - Which method would you use to estimate the parameters of the identified equation (s)? Why?
6. Explain whether the following statements are true or false. Give a brief explanation.
- In multiple regressions, a high correlation in the sample among the regressors implies that the least square estimators of the coefficient are biased.
 - Whether or not Multi-collinearity is a problem cannot be decided by just looking at the Inter-correlation between the explanatory variables.
 - If the coefficient estimates in an equation have high standard errors, this is evidence of high Multi-collinearity.
 - The relevant question to ask if there is high Multi-collinearity is not what variables to drop but what other information will help.

7. Consider the following regression equation:

$A_t = 44 + 18N_t$	$R^2 = 78$
(3) (6)	$n = 40$
	$d = 1.40$

Where

A_t = Number of accident on a highway during day t

N_t = Number of cars traveling the highway on day t

(The numbers in parenthesis are the standard errors)

- Does auto-correlation exist at 5 percent level of significance?
- Is the auto-correlation positive or negative?

8. Define the terms "Heteroscedasticity" and "Homoscedasticity". Explain the effects of "Heteroscedasticity" on the estimates of the parameters and their variances in a normal regression model.

1-a) Based on 76 observations the following regression to explain personal consumption expenditure (PCE) in Pakistan is obtained:

$$y = -10.96 + 0.93X_2 - 2.09X_3$$

$$t = (-3.33) \quad (1249.06) \quad (-3.09)$$

$$R^2 = 0.996, \quad F = 83753.7$$

Where Y = the PCE (Rs. In billions)

X_2 = disposable income (Rs. In billion)

X_3 = prime rate (%) charged by banks.

- What is the marginal propensity to consume (MPC)?
- Is the MPC statistically different from 1?
- What is the rationale for the inclusion of the prime rate variable in the model? A priori would you expect a negative sign for this variable?
- Is coefficient of X_3 significantly different from zero?

b) Estimate the PRF $y = a e^{bx+u}$. Also predict y for $x = 30$ Given:

X:	10	12	13	15	18	20	25
Y:	225	280	300	325	410	490	600

2. a) Given $b = \sum C_i y_i$, determine the weight C_i , which makes b a best linear unbiased estimator of the intercept in two variable model.

b) For certain data $\hat{Y} = 10.5 + 1.4X$ and $\hat{X} = 15.4 + 0.6Y$ are the two regression lines. Compute r and S_x/S_y . If $y = x + u$ compute the correlation between x and u .

3. a) A specification with four explanatory variables is estimated using twenty observations. Give residual sum of squares = 6788.4 and $R^2 = 0.92$

- Construct the ANOVA table and test the relevant hypotheses
- Determine the value of adjusted R-Square.

b) From the regression equation of Y on X and few other information given below, obtain the equation of X on Y .

$$\hat{Y} = 52.375 + 1.85 X$$

$$R^2 = 0.975, \quad \bar{Y} = 320.$$

c) Describe the use of qualitative explanatory variables, how intercept and slope dummy are incorporated.

4. Based on 9 observations given the following sums, sums of squares and product in deviation form. Where Y are indices of imports at constant prices, X_2 the indices of Gross Domestic prices and X_3 the deflated import price index.

$$\sum Y = 1,052; \quad \sum X_2 = 1,017$$

$$\sum X_3 = 954; \quad \sum Y^2 = 1260.89$$

$$\sum X_2 = 650; \quad \sum X_3^2 = 648$$

$$\sum x_2 Y = 874; \quad \sum x_3 Y = -70$$

$$\sum X_2 X_3 = -112$$

- Estimate the regression equation.
 - Estimate the standard errors of the slope coefficients.
 - Test the significance of these coefficients.
 - Determine the value of R-Square and the adjusted R-Square.
5. Show that the least square estimator of the B-Vector has minimum Variance. Also, obtain an unbiased estimator of the disturbance Variance. Explain all steps where necessary.

6. a) Describe and distinguish between the Dummy variable test for structural difference and the Chow test.

b) Given the following information about the two-subgroups of data and the pooled data.

Group	T.S.S	R2	# OBS
I	18233	0.9021	12
II	12619	0.2071	14
I+II	99863	0.7672	26

Use suitable statistic(s). Check for:

- Heteroscedasticity between the groups
- The Structural Difference.

7. a) An econometrician estimated a three variable model with a two variable specification. Examine the effects of this misspecification on this estimator.

b) Describe classical approach to obtain consistent estimators in the presence of contemporaneous correlation.

8. Write a detailed note on any one of the following.

- Auto-correlation
- Heteroscedasticity

1. From a data of 19-observations the following cross product matrix is obtained:

	Y	X ₁	X ₂
Y	14.94		
X ₁	14.33	17.24	
X ₂	14.55	17.19	23.39
Also	ΣY=12.1	ΣX ₁ =13.5	ΣX ₂ =20.5

(i) Estimate the coefficients of the following model:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \text{error}$$

(ii) Test the significance of β_1 and β_2 at $\alpha = 0.05$ and 0.01

(iii) Compute R^2 and comment on the suitability of model.

2. a) Describe and explain the Run Test for auto correlation of residuals.

b) The following table gives the residual obtained after applying the consumption function:

$$C = -3.0 + 0.9277y_a + e$$

Year	1	2	3	4	5	6	7	8	9	10	11	12
C	0.6	1.9	-1.8	-2.7	-2.9	1.4	3.3	0.03	0.8	2.3	-1.4	-1

Compute the Durbin-Weston test statistics and comment.

3. a) Explain the following terms in the light of least square assumptions.

(i) Randomness

(ii) Zero mean independent of X.

(iii) Homoscedasticity

(iv) Non-autocorrelation of errors

(v) Normality.

b) The following are the convenience matrices of errors. Differentiate them with respect to autocorrelation and homoscedasticity.

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 2 \end{pmatrix}$$

4. A study was conducted by retail merchants to determine the relation between weekly advertising expenditure and sales. The following data were recorded:

Advertising 40 20 25 30 50 40

Expenditure Sale 385 400 395 475 440 425

(i) Find the equation of regression to predict weekly sale from advertising expenditure.

(ii) Test the significance of slope (β) by constructing 95% confidence interval.

(iii) Project the sale value when advertising expenditure is 60.

(iv) Compute sum of squares of error.

5. Explain with examples how a non-linear model may be treated as linear in the estimation of parameters.

b) Derive the normal equation for determining the least square estimate of 'a' and 'b' of the linear model:

$$y_1 = a + b_1 + c_1$$

Also obtain the formula for the computation of 'a' and 'b'.

6. Write notes on any three of the following:

(i) Instrumental variable

(ii) Multi-collinearity

(iii) Chow test for structural difference

(iv) Dummy variable trap

(v) Goldfeld-Quandt test

7. a) Given $\hat{\beta}_1 = \sum w_1 y_1$ where $y_i = \beta_1 + \beta_2 x_1 + u_1$. Obtain the weight w_1 which makes $\hat{\beta}_1$ a BLUE.

b) Obtain an unbiased estimator of the standard error of estimate in a two variable model.

8. Describe the significance of distributed Log models. Explain the Koyck procedure.

b) How the values of $\hat{\beta}_1, \hat{\beta}_2, \text{var}(\hat{\beta}_1), \text{var}(\hat{\beta}_2), R^2$ and the t values would be effected due to a linear transformation of the type given below:

$$y^* = w_1 y \text{ and } x^* = w_2 x$$

1. a) Define and explain properties of Estimators for small sample and Large sample.
- b) Write down procedure for Testing H_0 and H_1 .
2. Discussing with the help of suitable examples the possible uses of dummy variables in
 - (i) Comparing two regressions.
 - (ii) Pooling times series with cross-section data.

3. Based on 76 observations the following regression to explain consumption expenditures in Pakistanis obtained.

$$Y = -10.96 + 0.93X_1 - 2.09X_2$$

$$t = (-3.33) (1249.06) (-3.09)$$

$$R^2 = 0.996$$

$$F = 83753.7$$

Y = Consumption expenditures in (Rs. in billion)

X_1 = Disposable Income (Rs in billion)

X_2 = Prime rate (%) charged by banks

1. What is the Marginal propensity to consume?
2. Is the MPC statistically different from 1?
3. Is coefficient of X_2 significantly different from
4. Define and explain Heteroscedasticity.
5. a) Show that the least square estimator of " β_2 " in $Y = \beta_1 + \beta_2X_2 + u$ is BLUE
- b) Develop a statistic to test the significance of an observed coefficient β_2 .
6. From the following information, estimate the questions. $Y = b_0 + b_1X_1 + b_2X_2 + e$

$\bar{Y} = 65$	$\tilde{S}_y^2 = 700$
$\bar{X}_1 = 19$	$SX_1^2 = 18$
$\bar{X}_2 = 52$	$SX_2^2 = 180$
$Sy_{X_1} = 20, \quad \Sigma y_{X_2} = 225, \quad \Sigma x_1x_2 = -33$	
$S.E(b_1) = 1.32$	

t-Table value is 2.571

7. Define and explain few Econometric problems.
8. " R^2 " as a measure of explained variance. Explain Arc LOW R^2 , is a problem? Why adjusted coefficient of determinations is used?